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Optimum Shape Design of Rotating Shaft by ESO Method

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Abstract

Evolutionary structural optimization (ESO) method is based on a simple idea that the optimal structure can be produced by gradually removing the ineffectively used material from the design domain. ESO seems to have some attractive features in engineering aspects: simple and fast. In this paper, ESO is applied to optimize shaft shape for the rotating machinery by introducing variable size of finite elements in optimization procedure. The goal of this optimization is to reduce total shaft weight and resonance magnification factor (*Q* factor), and to yield the critical speeds as far from the operating speed as possible. The constraints include restrictions on critical speed, unbalance response and bending stresses. Sensitivity analysis of the system parameters is also investigated. The results show that new ESO method can be efficiently used to optimize the shape of rotor shaft system with frequency and dynamic constraints.

Keywords: Evolutionary structural optimization; Rotating shaft; Resonance response; Critical speed; Q-factor

1. Introduction

In the design of modern turbomachinery, it is often necessary to improve the performance of rotor-bearing systems. This usually requires maximizing the volume flow rate and pressure, and minimizing the energy losses, through a machine of limited size and weight. This generally suggests high shaft speeds, multiple stages, highly loaded rotating components, large spacing between stages, etc. All of these features tend to create rotordynamic problems. (Vance, 1988) Since critical speed range influences performance and safety of the whole system, it may be necessary and better to constrain the critical speeds and the resonance magnification factor (*Q* factor) in the design process to avoid large vibration. And the minimization of response amplitudes within the operating speed range of the system may be the most primary design objective. The problem of weight minimization usually arises from the revision of an existing rotor-bearing system to promote the system performance. As performance considerations and rotordynamic considerations are often in conflict, the design of a successful and reliable machine requires cooperation and compromises between the two disciplines.

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Many papers have shown that the system parameters, including the distribution of the mass, stiffness of the shaft and the dynamic coefficients of the bearings, have an influence on the dynamic characteristics of a rotor-bearing system. (Rajan et al., 1987; Wang and Shih, 1990; Diewald and Nordmann, 1990; Doizelet and Bondoux, 1990; Shiau and Hwang, 1990; Shiau and Chang, 1993) These studies used gradient-based mathematical programming methods, such as sequential quadratic programming approach, made use of local curvature information derived from linearization of the original functions by using their derivatives with respect to the design variables. These methods present a satisfactory local rate of convergence, but they cannot assure that the global optimum can be found and also consume a large part of the total computational effort. On the other hand, the application of evolutionary algorithms based on probabilistic searching, such as genetic algorithm (GA) (Goldberg, 1989; Holland, 1975) and evolution strategies, (Rechenberg, 1973; Schwefel, 1981) can avoid performing the computationally expensive analysis step because they do not need gradient information. They may suffer, however, from a slow rate of convergence towards the global optimum and do not guarantee convergence to the global optimum. (Lagaros et al., 2002) Choi and Yang has studied the optimum shape design of a rotor-bearing system with restrictions on critical speeds using the GA (Choi and Yang, 2000) and immune-genetic algorithm. (Choi and Yang, 2001) These papers focuses on the optimum design of a rotor-bearing system with minimum shaft weight, minimum Q factor and enough separation margin of the critical speed under the requirements of dynamic behaviors such as dynamic stress and steady-state unbalance response, to increase the performance of a rotor-bearing system.

Evolutionary structural optimization (ESO) method, which is introduced by Xie and Steven (Xie and Steren, 1993) in 1993, is based on the simple idea that the optimal structure (maximum stiffness, minimum weight) can be produced by gradually removing the ineffectively used material from the design domain (Xie and Steren, 1997; Querin et al., 1998). ESO seems to have some attractive features in the engineering aspects. The ESO is very simple to program via the finite element analysis (FEA) package and requires a relatively small amount of FEA time. Additionally, the ESO topologies have been compared with analytical ones, and so far the results are quite promising. On the other hand, ESO does not have a solid theoretical basis, and consequently, the ESO minimization problem is still unsolved (Tanskanen, 2002). The primary goal of research and development of ESO is to provide engineering industry with a practical and user-friendly optimization method to assist in the design process. Hence, ESO has been extended to accommodate various optimization criteria and is becoming a more practical method. Some of these researches include the implementation of stiffness and displacements as optimization criteria (Zhao et al., 1996) and the applications in multiple loads (Xie and Steren, 1996), non-linear dynamic (Zhao et al., 1996) and buckling problems (Chu et al., 1996). Querin et al. extended the ESO method to add as well as remove elements, namely bi-directional ESO (BESO). This means that the initial design no longer had to be the maximum design domain. Thus the solution time may be reduced especially if the user specifies a near optimal topology to be the initial design. However, this knowledge is not always available and the typical long solution time of ESO has been an obstacle to its practical applicability as a design tool.

In conventional ESO method, the size of finite element in each iterative step is always fixed. To get a fine shape of optimum, the design model should be divided more detail, and it takes very long calculation time because FEA is executed in each iterative step. To overcome these demerits of ESO shown in previous works, a new method that is faster and more accurate scheme is investigated in this paper by introducing the variable size of finite element in each iterative step.

In this paper, as the design variable is the diameter of each shaft element, the finite element that should be removed or added is the type of shell which has unit width of direction to diameter. To applying this new approach, firstly, the eigenvalue and the sensitivity analysis of initial model is executed, and then calculate the sensitivity numbers of object function for the diameter of each element. And the sensitivity numbers of each element are compared, and add or remove the element (in this model, increase or decrease the diameter of element) in proportion to sensitivity numbers. As the iteration number increase, the size of element becomes more precisely and finally converge to optimum shape. The proposed ESO method is applied to find the optimum shape of a rotor shaft in the electric motor so that the optimized rotor-bearing system can yield the minimum shaft weight, Q factor and enough separation margin of critical speed with the dynamic behavior constraints. The results show that the proposed ESO algorithm can reduce weight of the shaft and Q factor, and yield the critical speeds as far from operating speed as possible with dynamic constraints.

2. Rotor model and theory

Usually the rotor-bearing system is modeled as an assemblage of the discrete bearings and the rotor segments with distributed mass and elasticity. In order to make an accurate analysis of the complex rotor-bearing systems, the vibration is calculated using general finite element procedures in this paper. Since the finite element discretization procedure is well documented in many literatures (Nelson and McVaugh, 1976), the details will be omitted here and only the equations of motion are presented below.

The system equations that describe the behavior of entire rotor-bearing system are formulated by taking into account the contributions from all elements in the model. The assembled equation of motion with N_e elements in the whirl frame coordinates is of the form (Choi and Yang, 2002)

$$\boldsymbol{M}\,\ddot{\boldsymbol{p}} + \boldsymbol{C}\,\dot{\boldsymbol{p}} + \boldsymbol{K}\,\boldsymbol{p} = \boldsymbol{Q}^u \tag{1}$$

where $\boldsymbol{M}$ (= $\boldsymbol{M}_t + \boldsymbol{M}_r$) is the mass matrix, $\boldsymbol{M}_t$, $\boldsymbol{M}_r$ are the translational and rotational mass matrices, $\boldsymbol{C}$ (= $-\Omega\,\boldsymbol{G} + \boldsymbol{C}_b$), $\boldsymbol{K}$ (= $\boldsymbol{K}_b + \boldsymbol{K}_s$) are the damping and stiffness matrices, $\boldsymbol{G}$ is a gyroscopic matrix, $\boldsymbol{K}_b$, $\boldsymbol{C}_b$ are the stiffness and damping matrices of bearing, and $\boldsymbol{Q}^u$ is a force vector, respectively.

2.1 Eigenvalue analysis

In setting up the complex eigenvalue problem for the whirl frequencies of the system governed by Eq. (1), it is convenient to write the system equation in the first order state vector form as

$$\boldsymbol{A}\dot{\boldsymbol{q}} + \boldsymbol{B}\boldsymbol{q} = \boldsymbol{0} \tag{2}$$

where

$$\boldsymbol{A} = \begin{bmatrix} \boldsymbol{C} & \boldsymbol{M} \\ \boldsymbol{M} & \boldsymbol{0} \end{bmatrix},\ \boldsymbol{B} = \begin{bmatrix} \boldsymbol{K} & \boldsymbol{0} \\ \boldsymbol{0} & \boldsymbol{-M} \end{bmatrix},\ \boldsymbol{q} = \begin{bmatrix} \dot{\boldsymbol{p}} \\ \boldsymbol{p} \end{bmatrix}$$

For assumed harmonic solution $\boldsymbol{q} = \bar{\boldsymbol{q}} e^{\lambda t}$ of Eq. (2), the associated eigenvalue problem is

$$(\boldsymbol{A}\lambda + \boldsymbol{B})\,\bar{\boldsymbol{q}} = \boldsymbol{0} \tag{3}$$

where λ is the eigenvalue. The eigenvalues are usually complex eigenvalues and conjugate roots

$$\lambda_i = \alpha_i \pm j\omega_i \tag{4}$$

where α_i, ω_i are the growth factor and the damped natural frequency of the ith mode, respectively. The Q factor in the critical speed (Q_i) is expressed in term of the real and imaginary parts of complex eigenvalue.

$$Q_i = \frac{1}{2\zeta_i} = -\frac{\sqrt{\alpha_i^2 + \omega_i^2}}{2\alpha_i} \tag{5}$$

where ς_i is the damping ratio of the ith mode.

2.2 Steady-state unbalance response

The mass unbalance forces $\boldsymbol{Q}^u$ shown in Eq. (1) can usually be expressed as follows:

$$\boldsymbol{Q}^u = \boldsymbol{Q}_0\,\Omega^2\,e^{j\Omega t} \tag{6}$$

where $\boldsymbol{Q}_0$ is independent of time and rotating speed. The steady-state response due to mass unbalance is assumed of the form

$$\boldsymbol{p} = \boldsymbol{p}_s\,e^{j\Omega t} \tag{7}$$

Substituting Eqs. (6) and (7) into Eq. (1) yields

$$(\boldsymbol{K} - \Omega^2(\boldsymbol{M} + \boldsymbol{N} - \boldsymbol{G}))\,\boldsymbol{p}_s = \Omega^2\,\boldsymbol{Q}_0 \tag{8}$$

Then the steady-state response can be obtained by solving Eq. (8) for $\boldsymbol{p}_s$.

2.3 Bending stress analysis

The bending stress resisted by the rotating flexible shaft can be obtained as follows [2]:

$$\sigma_{(l)}^{(i,m)} = \left[\sum_{j=1}^{4} K_{(2,j)}^{e^{(i)}} \cdot p_{s(j)}^{(i,m)}\right] r_{oi} / I_i \tag{9}$$

$$\sigma_{(r)}^{(i,m)} = \left[\sum_{j=1}^{4} K^{e(i)}_{(4,j)} \cdot p_{s(j)}^{(i,m)} \right] r_{oi} / I_i \tag{10}$$

where the superscript i denotes the ith shaft element; the superscript m denotes when $\Omega = \Omega_m$, Ω_m is associated with the operating speed; $\sigma_{(l)}^{(i,m)}$ and $\sigma_{(r)}^{(i,m)}$ represent the bending stress of a nodal point on the left-hand side and right-hand side of the shaft element; $p_{s(j)}^{(i,m)}$ represents the value of the $(4i-4+j)$th entry of the steady-state response vector $\boldsymbol{p}_s$ as $\Omega = \Omega_m$ and $K^{e(i)}_{(l,j)}$ denotes the value of the stiffness matrix $\boldsymbol{K}^e$ of the ith shaft element at the lth row and the jth column.

2.4 Sensitivity analysis

In the ESO method, a key issue is to evaluate the efficiency of material used in the design domain. For a static optimization problem, the efficiency of material can be evaluated by considering the stress level of each particular element. If the stress level of an element is very low, it means that the material of this element is not used efficiently and therefore can be removed. For the natural frequency optimization problem, there is not any external dynamic load in the system because the system is in a free vibration state. Thus, it is impossible to use stress level to determine the efficiency of material for an evolutionary natural frequency optimization problem. In order to solve this problem, it is essential to evaluate the individual contribution of an element to the natural frequency concerned since finite elements are basic cells of the design domain. Although the sensitivity of a natural frequency, which is usually expressed in the differentiation sense, can be used to evaluate the efficiency of an element, the contribution factor of each element to the natural frequency is used in this study to evaluate the efficiency of the element, because it is expressed in the difference sense and therefore may be most suitable to the finite element analysis.

2.4.1 Sensitivity analysis of eigenvalue

It can be expressed to Eq. (11) considering the ith mode complex eigenvalue and eigenvector from Eq. (2)

$$(\lambda_i \boldsymbol{A} + \boldsymbol{B})\, \boldsymbol{\varphi}_i = \boldsymbol{0} \tag{11}$$

where,

$$\boldsymbol{A} = \begin{bmatrix} \boldsymbol{C} & \boldsymbol{M} \\ \boldsymbol{M} & \boldsymbol{0} \end{bmatrix},\ \boldsymbol{B} = \begin{bmatrix} \boldsymbol{K} & \boldsymbol{0} \\ \boldsymbol{0} & \boldsymbol{-M} \end{bmatrix},\ \boldsymbol{\varphi}_i = \begin{bmatrix} \boldsymbol{\varphi}_{1i} \\ \lambda_i \boldsymbol{\varphi}_{1i} \end{bmatrix}$$

Taking the derivative of Eq. (11) with respect to design parameter d_j gives

$$\left(\frac{\partial \lambda_i}{\partial d_j} \boldsymbol{A} + \lambda_i \frac{\partial \boldsymbol{A}}{\partial d_j} + \frac{\partial \boldsymbol{B}}{\partial d_j} \right) \boldsymbol{\varphi}_i + (\lambda_i \boldsymbol{A} + \boldsymbol{B}) \frac{\partial \boldsymbol{\varphi}_i}{\partial d_j} = \boldsymbol{0} \tag{12}$$

Then premultiplying Eq. (12) by $\boldsymbol{\varphi}_i^T$

$$\frac{\partial \lambda_i}{\partial d_j} \boldsymbol{\varphi}_i^T \boldsymbol{A}\, \boldsymbol{\varphi}_i + \lambda_i \boldsymbol{\varphi}_i^T \frac{\partial \boldsymbol{A}}{\partial d_j} \boldsymbol{\varphi}_i + \boldsymbol{\varphi}_i^T \frac{\partial \boldsymbol{B}}{\partial d_j} \boldsymbol{\varphi}_i + \boldsymbol{\varphi}_i^T (\lambda_i \boldsymbol{A} + \boldsymbol{B}) \frac{\partial \boldsymbol{\varphi}_i}{\partial d_j} = \boldsymbol{0} \tag{13}$$

Solving Eq. (13) to obtain the eigenvalue sensitivity gives

$$\frac{\partial \lambda_i}{\partial d_j} = -\frac{\lambda_i \bar{p}_i' + \bar{q}'}{\bar{p}_i} \tag{14}$$

where,

$$\bar{p}_i = \boldsymbol{\varphi}_i^T \boldsymbol{A}\, \boldsymbol{\varphi}_i = \boldsymbol{\varphi}_{1i}^T \boldsymbol{C}\, \boldsymbol{\varphi}_{1i} + 2\lambda_i \boldsymbol{\varphi}_{1i}^T \boldsymbol{M}\, \boldsymbol{\varphi}_{1i},$$

$$\bar{p}_i' = \boldsymbol{\varphi}_i^T \frac{\partial \boldsymbol{A}}{\partial d_j} \boldsymbol{\varphi}_i = \boldsymbol{\varphi}_{1i}^T \frac{\partial \boldsymbol{C}}{\partial d_j} \boldsymbol{\varphi}_{1i} + 2\lambda_i \boldsymbol{\varphi}_{1i}^T \frac{\partial \boldsymbol{M}}{\partial d_j} \boldsymbol{\varphi}_{1i}$$

$$\bar{q}_i' = \boldsymbol{\varphi}_i^T \frac{\partial \boldsymbol{B}}{\partial d_j} \boldsymbol{\varphi}_i = \boldsymbol{\varphi}_{1i}^T \frac{\partial \boldsymbol{K}}{\partial d_j} \boldsymbol{\varphi}_{1i} + \lambda_i^2 \boldsymbol{\varphi}_{1i}^T \frac{\partial \boldsymbol{M}}{\partial d_j} \boldsymbol{\varphi}_{1i}$$

2.4.2 Sensitivity analysis of Q factor

Similarly by differentiating Eq. (5) with respect to design parameter d_j, the derivatives of Q factor can be obtained as

$$\frac{\partial}{\partial d_j} Q_i = \frac{\alpha_{ij}}{2\alpha_i^2} \sqrt{\alpha_i^2 + \omega_i^2} - \frac{\alpha_i \alpha_{ij} + \omega_i \omega_{ij}}{2\alpha_i \sqrt{\alpha_i^2 + \omega_i^2}} \tag{15}$$

where,

$$\alpha_{ij} = \frac{\partial \alpha_i}{\partial d_j} = \mathrm{Re}\left(\frac{\partial \lambda_i}{\partial d_j} \right),$$

$$\omega_{ij} = \frac{\partial \omega_i}{\partial d_j} = \mathrm{Im}\left(\frac{\partial \lambda_i}{\partial d_j} \right)$$

2.4.3 Sensitivity of weight

The total weight of shaft can be expressed as follows

$$W = \sum_{i=1}^{N_e} \rho l_i \pi \frac{(d_{Oi} - d_{Ii})^2}{4} \tag{16}$$

where l_i, d_{Oi} and d_{Ii} are the length, outer diameter and inner diameter of the ith element, ρ is the density of element and N_e is the total number of element, respectively.

As the outer diameter of each element is taken into design variable in this paper, the derivatives of other elements are zeros and finally the derivatives of total shaft weight for the jth element diameter are of the form

$$\frac{\partial W}{\partial d_j} = \frac{\rho l_j \pi}{2}(d_{Oj} - d_{Ij}) \tag{17}$$

3. Optimal design procedure

In the rotordynamic problem, since the rotating shaft has circular cross sections and is usually modeled by the beam finite element, the topology of shaft can be optimized by changing the diameters of each finite element gradually. Therefore the diameters of each element were selected as design variables.

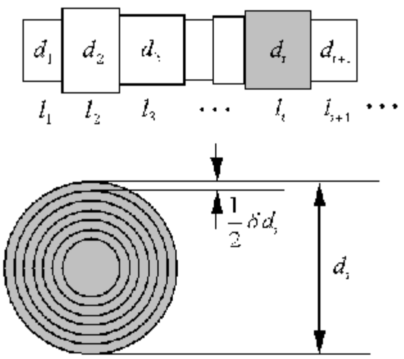

Fig. 1. The definition of design variables considered.

From the viewpoint of the traditional ESO method, the type of finite element that should be removed or added is a circular shell of unit thickness (δd_i). Figure 1 describes the design variables considered in the finite elements and the cross section of the shaft.

In this study, the objective function $F(\boldsymbol{x})$ is composed of the shaft weight $W(\boldsymbol{x})$, natural frequency $\omega_i(\boldsymbol{x})$ and Q factor $Q_i(\boldsymbol{x})$. For this example, the operating speed (3000 rpm) is between the 2nd forward natural frequency (2397 cpm) and the 3rd forward natural frequency (9071 cpm). The operating speed is near the 2nd natural frequency and the 3rd natural frequency is far from the operating speed sufficiently. So the objective function is as follows:

$$F(x) = \alpha \frac{W(x)}{W_0} + \beta \frac{\omega_{2F}(x)}{\omega_{20}} + \gamma \frac{Q_2(x)}{Q_{20}} \Rightarrow \textit{Minimize} \tag{18}$$

where α, β and γ are weighting factors. Each value of the objective function is divided by the reference value to make the objective function dimensionless and all value of three items having equal value range because three items in the objective function have a different unit and scale. The constraints on the bending stress and unbalance response are taken as follows:

$$g_1(\boldsymbol{x}) = |\sigma_{\max}| - \sigma^* \le 0$$
$$g_2(\boldsymbol{x}) = |\delta_{\max}| - \delta^* \le 0 \tag{19}$$

where $\sigma_{\max}$ and $\delta_{\max}$ denote the maximum bending stress and response in the steady-state. σ^* and δ^* represent the allowable stress and allowable steady-state response, respectively.

From Eqs. (14), (15), (17) and (18), the sensitivity number SN of objective function is defined as follows:

$$SN = \frac{\partial F}{\partial d_j} = \frac{\alpha}{W_0}\frac{\partial W}{\partial d_j} + \frac{\beta}{\omega_{20}}\frac{\partial \omega_{2F}}{\partial d_j} + \frac{\gamma}{Q_{20}}\frac{\partial Q_2}{\partial d_j} \tag{20}$$

Hence, the solution procedures using sensitivity number of multi-objective optimization problem are outlined as follows, and Fig. 2 describes the basic operation of ESO.

Step 1. FE analysis of new model which produces total weight W, natural frequency ω_{2F} and Q factor Q_2 and calculates the value of objective function $F(\boldsymbol{x})$

using Eq. (18).

Step 2. Sensitivity analysis which produces *SN* of objective function for each diameter of element using Eq. (20).

Step 3. Changing the diameter of each element in proportion to sensitivity number. If the *SN* of the *i*th element is positive, the diameter of this element has to be decreased, because the sensitivity number means the change of objective function per positive unit change of design variable and the goal is to minimize the objective function. Hence, it needs to determine how much change should be accomplished. In this paper, following changing criterion is used.

$$\delta d_i = \begin{cases} -\dfrac{SN_i}{SN_{\max}}\left(d_{i\max} - d_i\right)\ C_P & : SN_i < 0 \\ \dfrac{SN_i}{SN_{\max}}\left(d_i - d_{i\min}\right)\ C_P & : Otherwise \end{cases} \tag{21}$$

where, δd_i is the amount of change of the *i*th element, SN_i is the sensitivity number of the *i*th element, SN_{max} is the maximum absolute value of sensitivity number among all element, d_i is the diameter of the *i*th element, C_p is the changing rate of the shaft at each iterative step, $d_{i\max}$, $d_{i\min}$ are the maximum and minimum allowable diameter of the *i*th element at present iterative step (*j*th step).

If the sign of *SN* of this element in present iterative step (*j*th step) is changed from previous step, there must be the optimum value between present diameter and previous diameter. So the limit of diameter needs to be changed as follows:

$$\begin{cases} d_{i,j\max} = d_{i,j-1} \ : \ SN_{i,j}SN_{i,j-1} < 0,\ \ SN_{i,j} < 0 \\ d_{i,j\min} = d_{i,j-1} \ : \ SN_{i,j}SN_{i,j-1} < 0,\ \ SN_{i,j} \geq 0 \end{cases} \tag{22}$$

where, *i* and *j* denote the *i*th element and *j*th step of iteration, respectively.

Step 4. Checking the constraints condition. If all constraints are not satisfied, failed elements which make constraints unsatisfactory can be found by examining the constraints according to change in the diameter of each element. Then a new shaft shape can be obtained by changing the diameter of shaft based on the step 3 except failed elements.

Step 5. Repeat step 1 to step 3 until the following convergence criterion C_c is satisfied.

$$\frac{|\,F_{j-1} - F_j\,|}{F_{j-1}} \leq C_c \tag{23}$$

```
Input data of a rotor-bearing system
Define the objective function, design variables and side constraints
Define Cp and Cc
Main Procedure
    While (j = 1 to Max_iteration)
        call Objective_Function ( Return Obj_Fun(i, j) )
        call Sensitivity_Number ( Return SN(i, j) )
        While (i = 1 to No_of_design_variable)
            If (SN(i, j) < 0)    Then
                dia(i, j) = dia(i, j-1) - SN(i, j)/max(SN(all, j))*(max_dia(i) - dia(i, j))*Cp
                If (SN(i, j)*SN(i, j-1) < 0 )    Then
                    max_dia(i) = dia(i, j-1)
                End if
            Else
                dia(i, j) = dia(i, j-1) + SN(i, j)/max(SN(all, j))*(dia(i, j) - min_dia(i))*Cp
                If (SN(i, j)*SN(i, j-1) < 0 )    Then
                    min_dia(i) = dia(i, j-1)
                End if
            End if
        End while
        call Check_Constraints ( Return dia(i, j) )
        If (abs(Obj_Fun(j) - Obj_Fun(j-1)) / Obj_Fun(j-1) ) <= Cc )    Then
                Stop Main_Procedure
        End if
    End while
End Main_Procedure
```

Fig. 2. Optimization procedures of ESO.

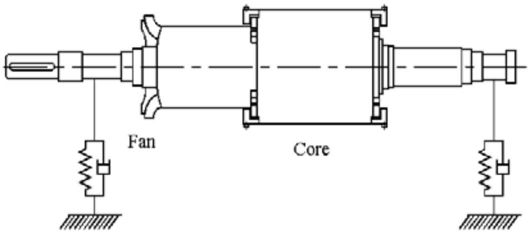

Fig. 3. The schematic of motor model (Choi and Yang, 2001).

Table 1. Configuration data of induction motor.

Motor	2 Pole, 50 Hz, 2200 kW
Rotor weight	13.105 kN
Core weight and Polar moment of inertia	7.67 kN, 276.5 N·m^2
Fan weight and Polar moment of inertia	0.177 kN, 7.85 N·m^2
Shaft	L = 2.847 m, W = 5.207 kN, E = 206 GN/m^2, G = 83.06 GN/m^2, γ = 77 kN/m^3
Bearing	2 lobe bearing (preload factor = 0.5) C = 0.1 mm, L = 150 mm, D = 125 mm

4. Numerical example

In order to illustrate how the ESO can be used to find the optimum value of a shaft diameter, a numerical example is presented. The example is taken from Choi and Yang (Choi and Yang, 2001) which is the three-phase induction motor for heavy duty as shown in Fig. 3. The principle data for this motor are listed in Table 1. In this example, ESO is used to minimize the shaft weight, Q factor and yield the critical speed so far from the operating speed as possible. The optimum model is compared with an original model in the unbalance response, Campbell diagram, deflection and bending stress.

The side constraints of the design variables are given by

$12.5\text{ mm} \le d_i \le 20.5\text{ mm}$, $i = 1$ to 19

$d_1 \sim d_4$, d_{10}, d_{11}, $d_{17} \sim d_{19}$: will not change

In Eq. (18), W_0 (= 5.27 kN), ω_{20} (= 255 rad/s) and Q_{20} (= 20.24) are original model value and $\alpha = \beta = 1$, $\gamma = 0.004$. The weighting factors of the shaft weight and the natural frequency are set to be same and weighting factor of Q factor is smaller than others, because Q factor is much influenced by bearing than shape of shaft elements. In ESO algorithm, the changing rate of shaft diameters at each iterative step is set to 55% of changeable region ($C_p = 0.55$) and the convergence criterion factor C_c is set to 5×10^{-5}. Figure 4 shows the shape of an original, intermediate and optimum shaft. The gray area shows fixed elements. It can be seen that which element is more effective to decrease the objective function. Figure 5 shows the change of shaft weight, 2nd natural frequency and objective function in each iterative step. Also, this figure represents that these parameters are converged within several times of iteration. In Figs. 6 and 7, the comparison of critical speeds and unbalance responses before and after optimization are shown. Compared with the original model, the 2nd critical speed decreased from 2397 cpm to 1967 cpm, so the operating speed is sufficiently away from resonance region. To analyze the unbalance response, the allowable residual unbalance is calculated on the basis of ISO 1940 G 6.3. The allowable residual unbalance obtained is divided into two and applied to both ends of a core. In Fig. 7, the unbalance response in the center of core is shown. The unbalance response of an optimum model (43.4 μm) in the operating speed is reduced about 11.5 μm compared to the original model (54.9 μm) in Fig. 7. Also the maximum unbalance response at the bearing satisfies the vibration limit value (50.8 μm) of API standards in bearing, sufficiently.

Figure 8 shows a comparison of the deflection and

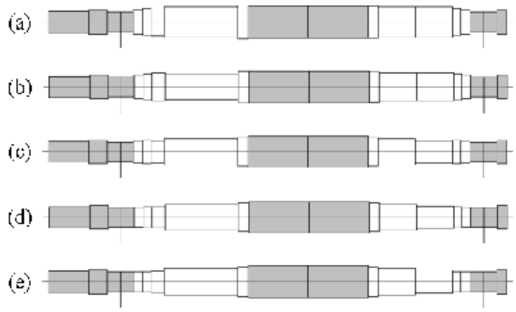

Fig. 4. Evolving shaft shape for a motor rotor, (a) Original design; (b) after 1st iteration; (c) after 3rd iterations; (d) after 5th iterations; e) optimum design (after 18th iterations).

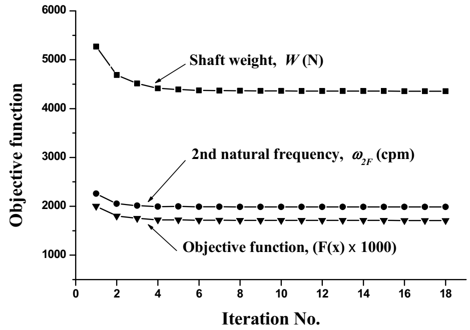

Fig. 5. Evolutionary history of the objective function.

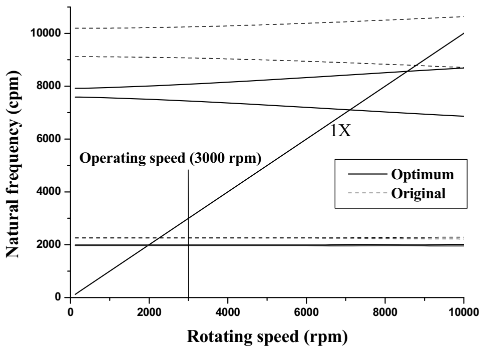

Fig. 6. Campbell diagram.

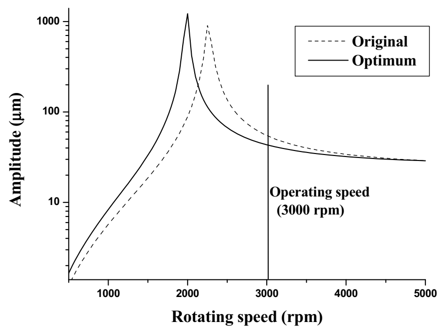

Fig. 7. Unbalance response at the mid-span of rotor core.

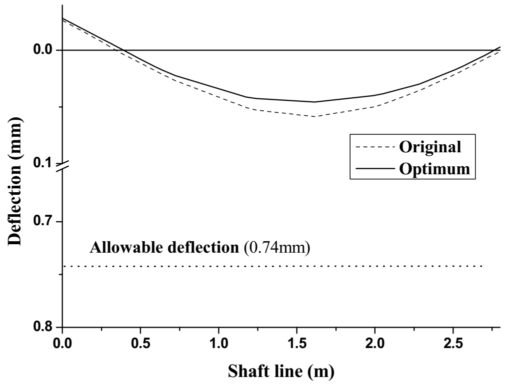

Fig. 8. Shaft deflection for the original model and optimum design.

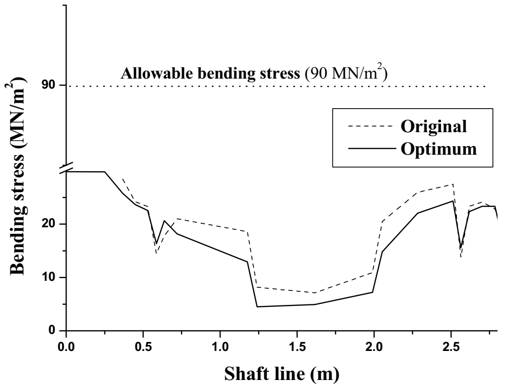

Fig. 9. Bending stress for the original model and optimum design.

Table 2. Comparisons of shaft diameters of the original and optimum design.

Element No.	Shaft element length l_i (m)	Shaft element diameter d_i (m)	
		Original design	Optimum design
1	0.250	0.1290	0.1290
2	0.117	0.1500	0.1500
3	0.081	0.1250	0.1250
4	0.081	0.1250	0.1250
5	0.057	0.1500	0.1250
6	0.051	0.1650	0.1251
7	0.083	0.1750	0.1252
8	0.457	0.1929	0.1524
9	0.063	0.2050	0.1718
10	0.375	0.1950	0.1950
11	0.375	0.1950	0.1950
12	0.063	0.1949	0.1686
13	0.230	0.1929	0.1552
14	0.230	0.1929	0.1308
15	0.051	0.1650	0.1251
16	0.055	0.1500	0.1250
17	0.083	0.1250	0.1250
18	0.083	0.1250	0.1250
19	0.062	0.1500	0.1500

Table 3. Total shaft weight, 2nd natural frequency, Q factor and unbalance response of the original model and optimum design.

Item	Original design	Optimum design
Total weight, $W(\boldsymbol{x})$	5.27 kN	4.19 kN
2nd natural frequency, $\omega_{2F}(\boldsymbol{x})$	2397 cpm	1967 cpm
Q factor, $Q_2(\boldsymbol{x})$	20.24	31.37
Unbalance response	54.9 μm	43.4 μm

bending stress before and after optimization. In general, the allowable deflection δ_w is calculated by

$$\delta_w \le 0.00033\, l \tag{24}$$

where l is the shaft length between two bearings. Comparing with an original model, the deflection is decreased a little in Fig. 8. Figure 9 shows a comparison of the bending stress before and after optimization. The maximum bending stresses of the original and optimum models are smaller than the allowable stress because the allowable bending stress is 90 MPa.

Tables 2 and 3 show the shaft diameter, total shaft weight, 2nd natural frequency, and Q factor of the original and optimum design.

5. Conclusions

Traditional ESO method has been applied successfully in structure topology optimization with its advantage of easy use and graphic-based implementation. This study applied the ESO method to optimum shape design of rotating shafts. As far as authors explored, this is the first attempt in the world. In the structural topology optimization using ESO method, often the objective is only to minimize the total weight, whereas in the shape optimization of rotating shaft, several design criteria and constraints are to be considered. The objectives for optimization in this study were set to minimize shaft weight and Q factor and to avoid the resonance region as far as possible from the operating speed under stress constraint and dynamic response constraint. To apply the structural ESO method to rotating shaft, sensitivities for each objective function were derived mathematically. The proposed method applied to optimum design of a large induction motor shaft. The results showed that total reliability of the system was greatly improved in terms of Q-factor, separation margin of critical speed and unbalance response, even with the reduced shaft weight. It is also noted that most rotating shafts in use are considered not to be optimized because the original shaft design used in this study is from a motor manufacturer. The proposed ESO method is expected to be used effectively in the design of rotating shafts in various industrial machineries as well as motors.

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